Box of Clay

Sample task from achievethecore.org
Task by Illustrative Mathematics, annotation by Student Achievement Partners

GRADE LEVEL Fifth

IN THE STANDARDS 5.MD.C

WHAT WE LIKE ABOUT THIS TASK

Mathematically:
- Centers on the fundamental volume concept of the unit cube (5.MD.C.3)
- Connects volume (5.MD.C) to previous multiplicative ideas of times-as-much

In the Classroom:
- Gives students the opportunity to develop the method they will use to solve the problem (drawing boxes, using snap cubes, applying the volume formula) (MP.5)
- Encourages students to talk about each other's thinking, in order to improve their mathematical understanding

MAKING THE SHIFTS[1]

Focus	Belongs to the major work[2] of fifth grade	
Coherence	Connects volume to multiplication and addition	
Rigor[3]	Conceptual Understanding: primary in this task	
	Procedural Skill and Fluency: secondary in this task	
	Application: primary in this task	

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Five.
[3]Tasks will often target only one aspect of rigor.

ADDITIONAL THOUGHTS

This task could be extended by adding a follow-up with larger numbers. That would enhance the connection to the fluency expectations in the Standards for multi-digit multiplication (5.NBT.B.5).

For more information about the expectations for volume in Grade 5, read pages 26 and 27 of the progression document, K-5 Geometric Measurement, available at
www.achievethecore.org/progressions.

5.MD Box of Clay

A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box has twice the height, three times the width, and the same length as the first box. How many grams of clay can it hold?

Commentary

This task provides an opportunity to compare the relative volumes of boxes in order to calculate the mass of clay required to fill them. These relative volumes can be calculated geometrically, filling the larger box with smaller boxes, or arithmetically using the given dimensions.

This purpose of this task is to help students understand what happens when you scale the dimensions of a right rectangular solid. At some point students need to understand that if you (for example) double the length, width, and height of a rectangular solid, then the volume increases by a factor of $2 \times 2 \times 2 = 8$ Before they get to the point of generalizing this phenomenon, they should think about the effects of scaling the different dimensions by different factors, as they do in this task.

This is a high-level instructional task for fifth graders, but it fits squarely within the scope of work that students are doing to understand the volume of right rectangular solids with whole-number side-lengths in fifth grade. Depending on how comfortable students are with visualizing rectangular solids, they might benefit from having snap-cubes or some other physical model to help them solve this problem.

This task was adapted from problem #3 on the 2012 American Mathematics Competition (AMC) 12A Test. The responses to the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage of Answers
(A)	120	2.08
(B)	160	3.60
(C)	200	5.15
(D)*	240	83.82
(E)	280	0.99
Omit	--	4.33

Of the 72,238 students who participated, 28,268 or 39% were in 12th grade, 34,124 or 47% were in 11th grade, 4,615 or 6% were in 10th grade, and the remainder were below 10th grade.

Solution: 1 Geometric visualization

The second box has 3 times the width and the same length as the first, smaller box. So we can fit three of the smaller boxes inside the second box to make one layer which will be 2 cm high. The second box is 2 times as high as the smaller one so we can add one more layer of three smaller boxes to fill the second box.

This means that it takes 6 small boxes to fill the large box so the large box holds six times as much as the small box. Since the small box holds 40 grams of clay, the large box holds $6 \times 40 = 240$ grams fo clay.

Solution: 2 Arithmetic comparison of volumes

The first box is 2 centimeters high, 3 centimeters wide, and 5 centimeters long so it has volume

$$2\text{cm} \times 3\text{cm} \times 5\text{cm} = 30 \text{ cubic centimeters}$$

and it holds 40 grams of clay. The second box is 4 centimeters high, 9 centimeters wide, and 5 centimeters long so its volume is

$$4\text{cm} \times 9\text{cm} \times 5\text{cm} = 180 \text{ cubic centimeters.}$$

Since the volume of the second box is $180 \div 30 = 6$ times bigger, it can hold 6 times as much clay. So the second box can hold $6 \times 40 = 240$ grams of clay.