

ACHIEVE THE CORE

Multi-Digit Multiplication Using the Standard Algorithm 5.NBT.B.5 Fluency Mini-Assessment by Student Achievement Partners

OVERVIEW

This mini-assessment is designed to illustrate the standard 5.NBT.B.5, which sets an expectation for fluently multiplying multi-digit whole numbers using the standard algorithm. This mini-assessment is designed for teachers to use either in the classroom, for self-learning, or in professional development settings to:

- Evaluate students' progress toward the skills described by 5.NBT.B.5 in order to prepare to teach this material or to check for fluency near the end of the grade;
- Gain a better understanding of assessing fluency with multiplying multi-digit whole numbers; and
- Illustrate CCSS-aligned assessment problems.

MAKING THE SHIFTS

This mini-assessment attends to **focus** as it addresses multi-digit multiplication, which is at the heart of the grade 5 standards and a key component of the major work of the grade.¹ It addresses **coherence** across grades because multiplying multi-digit whole numbers sets the stage for multi-digit division and multi-digit decimal operations. Standard 5.NBT.B.5 and this mini-assessment target *procedural skill and fluency*, one of the three elements of **rigor**.

A CLOSER LOOK

Standard 5.NBT.B.5 is a prime example of how "[t]he Standards are not written at uniform grain size" (K–8 Publishers' Criteria, Spring 2013, p. 18). One cannot address this standard in a single day, lesson, or unit. It will take significant classroom time throughout grade 5 for all students to master this standard.

5.NBT.B.5.
Fluently multiply multi-digit whole numbers using the standard algorithm.

The standard calls for fluency using the standard algorithm.

Standard algorithms for base-ten multiplication are based on decomposing numbers written in base-ten notation. This allows multiplication of two multi-digit whole numbers to be reduced to a collection of single-digit computations.² In the example below, in order to multiply 378×6 , one decomposes 378 to 3 hundreds, 7 tens, and 8 ones. Then, one multiplies 3×6 , 7×6 , and 8×6 .³ The diagram shows a variety of ways to help students think about how the standard algorithm works. By the end of grade 5, students should no longer need to build the standard algorithm, but should be employing it to perform fast and accurate multi-digit multiplication. Students who leave grade 5 without meeting standard 5.NBT.B.5 are likely to have difficulty fluently dividing multi-digit numbers using the standard algorithm (6.NS.B.2) and may have trouble with addition, subtraction, multiplication, and division of multi-digit decimals (6.NS.B.3).

Building the Standard Algorithm, 1-digit \times 3-digit:

$$378 \times 6$$

Area model

| | | | | | | |
|---|------------------------------------------------------------|---|---------------------------------------------------|---|---------------------------|------------------------------------------------------------------|
| | 300 | + | 70 | + | 8 | |
| 6 | $6 \times 300 =$ 6 \times 3 hundreds = 18 hundreds | | $6 \times 70 =$ 6 \times 7 tens = 42 tens | | $6 \times 8 =$ 48 ones | = 18 hundreds + 42 tens + 48 ones = 1800 + 420 + 48 = 2268 |

| | |
|------------|-----------------------|
| 378 | Left to right with |
| $\times 6$ | partial products |
| 1800 | 6 \times 3 hundreds |
| 420 | 6 \times 7 tens |
| 48 | 6 \times 8 ones |
| 2268 | |

| | |
|------------|-----------------------|
| 378 | Right to left with |
| $\times 6$ | partial products |
| 48 | 6 \times 8 ones |
| 420 | 6 \times 7 tens |
| 1800 | 6 \times 3 hundreds |
| 2268 | |

| | |
|------------|-----------------|
| 378 | Right to left |
| $\times 6$ | with regrouping |
| 144 | |
| 828 | |
| 2268 | |

¹ For more on the Major Work of the grade, see [achievethecore.org/emphases](http://www.achievethecore.org/emphases).

² For more on the standard algorithm, read page 3 of the progression document, *K–5, Number and Operations in Base Ten*.

³ For more examples like this, read *Standard Algorithms in the Common Core State Standards*.

ACHIEVE THE CORE

The problems on this mini-assessment use a variety of both size and type of factors. Problems range from 3-digit by 1-digit multiplication to 4-digit by 3-digit multiplication, allowing teachers to gather useful data for all students. There are also factors that contain 0s in order to assess if students can correctly account for a 0 in a factor.

Computation Algorithm:

A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Computation Strategy:

Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Elementary-grades students build up a repertoire of computation algorithms as well as computation strategies (see text box)⁴. For some computations, the standard algorithm is best; for other computations, opportunistic strategies are best. For example, the difference $612 - 13$ is easy to find by using a mental strategy (such as subtracting 12 then subtracting 1 more, or simply reasoning that the answer will be one less than 600). On the other hand, the difference $661 - 237$ is easy to find using the standard algorithm. Some of the products on this mini-assessment are best calculated algorithmically on paper, while others are best performed mentally or on paper using an opportunistic strategy. For this mini-assessment, examples of opportunistic strategies appear on page 6.

⁴ Definitions originally appear in the Glossary of the Common Core State Standards for Mathematics, http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf#page=85.

Multi-Digit Multiplication Mini-Assessment

Name: _____ Date: _____

1. $49 \times 975 =$ _____

2.
$$\begin{array}{r} 6,751 \\ \times 609 \\ \hline \end{array}$$

3. What is the product of 9 and 740?

4. Multiply 28 and 5,555.

5.
$$\begin{array}{r} 5,087 \\ \times 245 \\ \hline \end{array}$$

6. _____ $= 777 \times 777$

5.NBT.B.5 Fluency Mini-Assessment – Multi-Digit Multiplication Using the Standard Algorithm Answer Key

Solutions

- | | |
|--------------|--------------|
| 1. 47,775 | 2. 4,111,359 |
| 3. 6,660 | 4. 155,540 |
| 5. 1,246,315 | 6. 603,729 |

Diagnosing Incorrect Answers

The multiplication algorithm has many steps, which means many opportunities for students to make mistakes. For problems with a larger number of factors, a common difficulty for students is keeping work organized. Graph paper can help students keep their work neat and legible when it is time to add several partial products. Here are some tips on how to quickly look at incorrect responses and diagnose common errors:

- Issues of place value.** If answers are off by a factor of 10 or 100, then place value error is likely. It's a challenge for students to keep track of so many digits at once. For example, students commonly make place value mistakes finding the partial products for the digits in the tens and larger places in the multiplier.

Example of place value error (#5)

| | |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 5,087 \\ \times 245 \\ \hline 25435 \\ \mathbf{20348} \\ + \mathbf{10174} \\ \hline 55,957 \end{array}$ | <p>→ Student correctly multiplies 5,087 by 5.</p> <p>→ Student incorrectly multiplies 5,087 by 4 instead of 40.</p> <p>→ Student incorrectly multiplies 5,087 by 2 instead of 200.</p> |
| <p>The two incorrect partial products are bolded. The error in this case is that the student entirely ignored the place value of the digits in the multiplier, effectively multiplying by 5, 4, and 2 instead of by 5, 40, and 200.</p> | |

Below are two ways to correct these errors.

| Correcting place value errors with the distributive property | Correcting place value errors by clarifying place values | | | | | | | | | | | | |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|-------------|---------------------------------------------------------------------|---------------|-----------|-------------------------------------------------------------------|---------------|------------|--------------------|--|--|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 5,087 \\ \times 245 \rightarrow 200 + 40 + 5 \\ \hline \end{array}$ | $\begin{array}{r} 5,087 \\ \times 245 \\ \hline 25435 \text{ ones} \rightarrow 25,435 \\ 20348 \text{ tens} \rightarrow 203,480 \\ +10174 \text{ hundreds} \rightarrow +1,017,400 \\ \hline 1,246,315 \end{array}$ | | | | | | | | | | | | |
| <table border="0"> <tr> <td style="text-align: center;"> $\begin{array}{r} 5,087 \\ \times 200 \\ \hline 1,017,400 \end{array}$ </td> <td style="vertical-align: middle;"> \rightarrow </td> <td style="text-align: center;"> $1,017,400$ </td> </tr> <tr> <td style="text-align: center;"> $\begin{array}{r} 5,087 \\ \times 40 \\ \hline 203,480 \end{array}$ </td> <td style="vertical-align: middle;"> \rightarrow </td> <td style="text-align: center;"> $203,480$ </td> </tr> <tr> <td style="text-align: center;"> $\begin{array}{r} 5,087 \\ \times 5 \\ \hline 25,435 \end{array}$ </td> <td style="vertical-align: middle;"> \rightarrow </td> <td style="text-align: center;"> $+ 25,435$ </td> </tr> <tr> <td colspan="3" style="text-align: center;"> $\hline 1,246,315$ </td> </tr> </table> | $\begin{array}{r} 5,087 \\ \times 200 \\ \hline 1,017,400 \end{array}$ | \rightarrow | $1,017,400$ | $\begin{array}{r} 5,087 \\ \times 40 \\ \hline 203,480 \end{array}$ | \rightarrow | $203,480$ | $\begin{array}{r} 5,087 \\ \times 5 \\ \hline 25,435 \end{array}$ | \rightarrow | $+ 25,435$ | $\hline 1,246,315$ | | | <p>Being explicit with the place value of each partial product helps students understand the meaning behind the steps in multi-digit multiplication.</p> |
| $\begin{array}{r} 5,087 \\ \times 200 \\ \hline 1,017,400 \end{array}$ | \rightarrow | $1,017,400$ | | | | | | | | | | | |
| $\begin{array}{r} 5,087 \\ \times 40 \\ \hline 203,480 \end{array}$ | \rightarrow | $203,480$ | | | | | | | | | | | |
| $\begin{array}{r} 5,087 \\ \times 5 \\ \hline 25,435 \end{array}$ | \rightarrow | $+ 25,435$ | | | | | | | | | | | |
| $\hline 1,246,315$ | | | | | | | | | | | | | |

5.NBT.B.5 Fluency Mini-Assessment – Multi-Digit Multiplication Using the Standard Algorithm

Answer Key

2. **Issues of regrouping.** Closely related to place value issues is when students simply do not regroup, or “carry,” when finding partial products. Essentially, when completing the series of single-digit computations, students record the digit from the ones place and ignore the digit in the tens place that should be regrouped. The error is paired with the correct work below.

Example of error with regrouping (#6)

| | |
|-----------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 777 \\ \times 777 \\ \hline 4,999 \\ 49,990 \\ + 499,900 \\ \hline 554,889 \end{array}$ | <p>The student multiplies 7×7, ignoring the regrouped 4 in all but the largest place value for each partial product.</p> |
|-----------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|

Correct solution

| |
|-----------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 777 \\ \times 777 \\ \hline 5,439 \\ 54,390 \\ + 543,900 \\ \hline 603,729 \end{array}$ |
|-----------------------------------------------------------------------------------------------------------|

3. **Issues with factors including 0.** Questions 2, 3, and 5 have 0 as a digit in one of the factors. Answers that are about one-tenth as large as the correct solution may indicate issues with dealing with a 0 in a factor. The error is paired with the correct work below.

Example of error with 0 in multiplicand (#5)

| | |
|-------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 5,087 \\ \times 245 \\ \hline 2,935 \\ 23,480 \\ + 117,400 \\ \hline 143,815 \end{array}$ | <p>The student ignores the 0 in the multiplicand, effectively multiplying 587×245</p> |
|-------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|

Correct solution

| |
|-------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 5,087 \\ \times 245 \\ \hline 25,435 \\ 203,480 \\ + 1,017,400 \\ \hline 1,246,315 \end{array}$ |
|-------------------------------------------------------------------------------------------------------------------|

Example of error with 0 in multiplier (#2)

| | |
|----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 6,751 \\ \times 609 \\ \hline 60,759 \\ + 405,060 \\ \hline 465,819 \end{array}$ | <p>The student ignores the 0 in the multiplier, effectively multiplying $6,751 \times 69$.</p> |
|----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|

Correct solution

| |
|-------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 6,751 \\ \times 609 \\ \hline 60,759 \\ 000,000 \\ + 4,050,600 \\ \hline 4,111,359 \end{array}$ |
|-------------------------------------------------------------------------------------------------------------------|

4. **Issues with incorrect fact recall.** If answers are near the correct solution, then students may be making calculation errors based on lack of fluency with single-digit multiplication and division facts. The common result of an addition error is that a single digit of the answer is off by one. Multiplication errors, however, result in a greater variety of incorrect responses. Explicit reminders to check the addition and multiplication facts may help students be more precise in their calculations. Similarly, students may need additional work with developing basic fact fluency through strategic teaching of multiplication facts using a variety of strategies.

5.NBT.B.5 Fluency Mini-Assessment – Multi-Digit Multiplication Using the Standard Algorithm Answer Key

Opportunistic Strategies

There are several opportunities on this mini-assessment for students to use opportunistic strategies in conjunction with the standard algorithm. For example, in #4, the question is worded as a statement allowing students to set up the multiplication however they choose. Students who notice that multiplying by 5,555 will result in four very similar partial products may use an opportunistic strategy of placing 5,555 in the multiplier spot, rather than the multiplicand. The diagram below compares the two strategies.

Traditional approach with larger factor as the multiplicand

$$\begin{array}{r} 5,555 \\ \times 28 \\ \hline 44,440 \\ + 111,100 \\ \hline 155,540 \end{array}$$

Opportunistic strategy using the larger factor as the multiplier

$$\begin{array}{r} 28 \\ \times 5,555 \\ \hline 140 \\ 1,400 \\ 14,000 \\ + 140,000 \\ \hline 155,540 \end{array}$$

Another example allowing students to use an opportunistic strategy is in #3, multiplication by 9. A student may quickly multiply by 10 and subtract 740, rather than writing out the full multiplication.

Traditional approach

$$\begin{array}{r} 3 \\ 740 \\ \times 9 \\ \hline 6,660 \end{array}$$

Opportunistic strategy using 10

$$\begin{array}{r} 740 \\ \times 10 \\ \hline 7,400 \\ - 740 \\ \hline 6,660 \end{array}$$