Common Core Knowledge and Practice Survey Report Companion

Math High School Grades

About

The Common Core Knowledge and Practice Survey (Survey) is a tool for educators to use to reflect on their instructional practice and understanding of the Common Core State Standards (CCSS). Designed for use in a professional learning community (PLC) setting within a school, the Survey is meant to spark conversation, identify areas for growth, and offer concrete ways for teams of teachers to continue to align their practice to the Shifts. Following the administration of the Survey in a PLC, this resource can be used by coaches and teachers to better understand the Survey questions and their relationship to the Shifts.

The Survey was intentionally designed to take no more than 30 minutes. As a result, there are a limited number of questions pertaining to each aspect of the Shifts. When considering the performance of an individual or group on a Shift, it is important to look closely at the questions to tease out which aspect(s) of the Shift may be creating confusion. This resource is broken into two parts.

PART 1 : SURVEY

The Survey uses a variety of question types and formats to highlight specific features of the Standards and Shifts. It is intended to capture information from educators about their understanding of different elements of the Standards and how the Shifts manifest in planning and classroom instruction. Each Survey question will be followed by commentary. The "Purpose" section will offer information about why the question was included in the Survey. The "Rationale" section will explain the different response options.

Some of the Survey questions have correct answers because they ask about items that can be objectively verified. Other questions ask participants to apply their knowledge of the Shifts and Standards to a specific example or classroom scenario. For these questions, there may be circumstances in the course of instruction where any number of answer choices could be correctly applied; however, for the purpose of this Survey the correct answers are those actions that most closely tie instruction practice to the expectations of the Standards and Shifts. Finally, a subset of the questions about practice are designed to help illuminate what's currently happening in classrooms and schools, in order to facilitate conversation about what training or structures might support future work.

Each question will have a label to indicate which category the question relates to – one of the Shifts (Focus, Coherence, Rigor) or "Practice-Content Connections." Questions that have one or more correct answers will have the correct response(s) indicated.

Shift of Focus

Focus strongly where the Standards focus

Shift of Coherence

Think across grades, and link to major topics within grades

Shift of Rigor

In major topics pursue: conceptual understanding, procedural skill and fluency, and application with equal intensity.

Practice-Content Connections

Knowledge of the Mathematical Practice Standards and strategic connection of the Practice and Content standards in materials and instruction

PART 2 : SHIFTS SUMMARY

Part two shows which questions relate to each category. Part two also includes discussion questions and resource recommendations intended to drive conversation and provide a starting point for next steps and long-term professional learning..

Part 1 : Survey

Q. 4
Shift of Focus

Question
Which of the following belongs to the Major Work of the indicated grade or course? Please select all that apply.

a. 8th grade
- ❒ a) Represent and analyze quantitative relationships between dependent and independent variables
- ❒ b) Define, evaluate, and compare functions
- ❒ c) Understand and apply the Pythagorean Theorem
- ❒ d) Understand and calculate probability of single events
- ❒ e) I don't know

b. Typical 9th grade course (e.g., Algebra I)
- ❒ a) Quadratic inequalities
- ❒ b) Examine transformations on the coordinate plane
- ❒ c) Linear and quadratic functions
- ❒ d) Create equations to model situations
- ❒ e) I don't know

c. Typical 11th grade course (e.g., Algebra II)
- ❒ a) Exponential and logarithmic functions
- ❒ b) Polar coordinates
- ❒ c) Using functions to model situations
- ❒ d) Identify the measures of central tendency and distribution
- ❒ e) I don't know

Correct Answer - 8th Grade: b, c; Typical 9th grade course: c, d; Typical 11th grade course: a, c

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PURPOSE
Rather than racing to cover topics in a mile-wide, inch-deep curriculum, the Standards require us to significantly narrow and deepen the way time and energy is spent in the math classroom. By focusing deeply on the Major Work of each grade, students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom. The Standards are not separated into grades in high school in order to allow for a variety of course structures. Therefore, Major Work in high school consists of specific clusters and standards across the high school domains with relatively wide applicability across a range of postsecondary work. This question addresses teachers' knowledge of the Major Work of the grade and these Widely Applicable Prerequisites. More information can be found at www.achievethecore.org/focus.

RATIONALE
This question showed different grade levels depending on the grade the teacher selected at the beginning of the Survey. The selected grade, one grade level below, and one grade level above were displayed for this question.

8th grade
a) Represent and analyze quantitative relationships between dependent and independent variables - 6th grade standard

b) Define, evaluate, and compare functions - Major Work of 8th grade
c) Understand and apply the Pythagorean Theorem - Major Work of 8th grade
d) Understand and calculate probability of single events - 7th grade standard
e) I don't know.

Typical 9th grade course (e.g., Algebra I)
a) Quadratic inequalities - content likely to be in a more advanced HS course
b) Examine transformations on the coordinate plane - 8th grade standard
c) Linear and quadratic functions - Widely Applicable Prerequisite for HS
d) Create equations to model situations - Widely Applicable Prerequisite for HS
e) I don't know.

Typical 11th grade course (e.g., Algebra II)
a) Exponential and logarithmic functions - Widely Applicable Prerequisite for HS
b) Polar coordinates - not in the Standards
c) Using functions to model situations - Widely Applicable Prerequisite for HS
d) Identify the measures of central tendency and distribution - not in the Standards
e) I don't know.

Q. 5

Shift of Coherence

Question
Over the past school year, how frequently have you done the following?

	Almost never/Never	About once a month	Several times a month	About weekly	Several times a week
a) Discussed Common Core State Standards for Mathematics with teachers in other grades					
b) Discussed Common Core State Standards for Mathematics with teachers in your own grade					
c) Looked at student mathematical work with other teachers for the purposes of professional development					
d) Received suggestions from colleagues for curricular materials aligned to the Common Core State Standards for Mathematics					

Correct Answers - N/A

PURPOSE

Supportive environments in which teachers are able to discuss, question, and work collectively to study student expectations support a deeper understanding of the coherence of the Standards. Teacher collaboration can facilitate more consistency in both instructional materials and instruction, and allows teachers to see connections across grades more clearly and make those connections in their classrooms. This question addresses the frequency of teacher collaboration around the Standards.

RATIONALE

There is no correct amount of time required for teachers to spend working together; however, teacher collaboration can support high quality CCSS-aligned instruction.

a) Vertical collaboration helps teachers develop a deeper understanding of the expectations of their own grade and provides an understanding of the progression of the standards in the surrounding grades.
b) Teachers collaborating within a grade allows for a collective reflection on specific content standards, lesson, and activities that are applicable to the content of the grade.
c) Examining student work with other teachers is useful in identifying key misconceptions and discussing how to make classroom adjustments.
d) Finding and sharing aligned resources with colleagues is useful for many schools/districts in which instructional materials are not aligned to the Standards.

Q. 6

Shift of Rigor

Question

Carefully examine each standard below and select which aspect of rigor is being targeted.

	Conceptual Understanding	Procedural Skill and Fluency	Application	I don't know
a) Rewrite expressions involving radicals and rational exponents using the properties of exponents.				
b) Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).				
c) Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*				
d) Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.				
e) Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*				

Correct Answers -a) Procedural Skill and Fluency, b) Conceptual Understanding, c) Application, d) Procedural Skill and Fluency, e) Application

PURPOSE

In order to reach the depth of the Standards, a balance of conceptual understanding, procedural skill/fluency, and application is required in the classroom. Each standard embodies at least one aspect of rigor; often the language in the standard indicates which aspect is being targeted (e.g., "fluently" identifies a procedural standard; "understand" identifies a conceptual understanding standard, and "real world problems" identifies an application standard). This question asks teachers to identify the aspect of rigor in the standard; understanding the intended aspect of rigor in a particular standard is critical for teachers to plan and execute lessons that address the expected learning of the Standards.

RATIONALE

a) "Rewrite expressions involving radicals and rational exponents" requires students to manipulate algebraic expressions; this standard targets procedural skill/fluency.
b) "Understand that a function from one set… to another set assigns to each element of the domain exactly one element of the range" indicates students are learning the concept of a function; this standard targets conceptual understanding.
c) "Create equations and inequalities in one variable and use them to solve problems…" asks students to use the equations and inequalities they created to solve problems; this standard targets application.
d) "Solve linear equations and inequalities in one variable" targets procedural skill/fluency.
e) "Use proportional relationships to solve multistep ratio and percent problems" indicates students are applying their understanding of proportional relationships to a real-world context; this standard targets application.

Q. 7

Shifts of Focus, Coherence and Rigor

Question

Please indicate the extent to which you agree or disagree with the following statements as they relate to your mathematics teaching this school year.

	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree	Not Applicable
a) I choose which standards to teach based on the major work of the grade.					
b) I try to incorporate conceptual understanding into every lesson that I teach.					
c) I try to incorporate real-world applications into every lesson that I teach.					
d) I use the textbook to determine the order of the standards that I teach.					
e) I use the wording of the standards to determine if procedural skills, conceptual understanding, and/or real-world applications are emphasized in my lessons.					
f) I order lessons based on the order of the standards at my grade.					
g) I spend less time in the classroom on additional/supporting standards.					
h) I organize which standards to teach based on how they connect to one another within and across units.					
i) I seek to balance my unit across procedural skills, conceptual knowledge and real-world applications.					
j) I consider students' prior knowledge when writing my lesson and learning objective.					
k) I try to give equal importance to all topics throughout the year.					

Correct Answers - N/A

PURPOSE

There are many pedagogical choices a teacher can make when supporting students in mathematics. This question details an array of instructional practices and asks teachers to identify which behaviors they exhibit in the classroom. Given the variety of decisions that can be made in different educational contexts, many of the statements above could be valid. However, some instructional practices better support Standards-based instruction and a teacher fully implementing the Standards would be more likely to emphasize those practices over others. To learn more about the Shifts, go to www.achievethecore.org/shifts-mathematics.

RATIONALE

The practices associated with the italicized sections below are those that a teacher fully implementing the Standards is likely to emphasize.

a) *Major Work of the grade (and Widely Applicable Prerequisites) is prioritized content within the Standards and requires more class time. Making instructional decisions based on the Major Work of the grade is important for ensuring that students spend the time necessary to master the concepts and skills that lead to algebra.*
b) Conceptual understanding is one of the three aspects of rigor and should be incorporated into lessons appropriately – based on the expectation of the standard. Not all standards focus on the development of mathematical concepts; lessons based on standards that set expectations of fluencies or the application of math should not target conceptual understanding.
c) Application is one of the three aspects of rigor and should be incorporated to lessons appropriately – based on the expectation of the standard. Not all standards focus on applying mathematical concepts; lessons based on standards that set expectations of fluencies or building mathematical concepts should not target application.
d) While the textbook might present a sound mathematical sequence of topics, the curriculum should be carefully reviewed for alignment to the Shifts. Tools such as the Instructional Materials Evaluation Tool (IMET) can help determine whether adjustments should be made. For more information go to www.achievethecore.org/imet.
e) *The Standards require a balance of conceptual understanding, procedural skill/fluency, and real-world application across the year, but the realization of these in the classroom should be specific to the intent of the standard being addressed. As noted in the rationale for Q6, the language of a standard indicates the aspect(s) of rigor targeted by the standard. Each of the aspects of rigor does not need to be attended to every day or in every lesson. For more information go to www.achievethecore.org/rigor.*
f) The standards within each grade are organized by domain, not in a suggested instructional sequence. Decisions around how to sequence topics should be made thoughtfully based on the coherence of the content of the grade and the progressions of mathematical ideas.
g) *Teachers should spend the majority of instructional time on the Major Work of the grade and Widely Applicable Prerequisites, and strategically connect Additional/Supporting Work to strengthen an understanding of the Major Work. For more information go to www.achievethecore.org/focus.*
h) *The Standards are built on mathematical progressions that support the development of concepts and skills across grades.*
i) The three aspects of rigor should be balanced across a full year in order to ensure students master the full depth of the Shifts. Units might have a balance of the aspects of rigor, but the relative time on each aspect will depend on which standards are being addressed in that unit.
j) *Building upon students' prior knowledge helps to make connections across grades clear to students, which, in turn, strengthens student understanding. For more information go to www.achievethecore.org/coherence.*
k) *The Shift of Focus highlights that some content is more important for students to succeed and progress towards algebra. Teachers should spend the majority of instructional time on the Major Work of the grade, and strategically connect Supporting Work to strengthen an understanding of the Major Work. For more information go to www.achievethecore.org/focus.*

Q. 8

Shift of Coherence

Question

Consider this standard.

Solve linear equations in one variable.

i. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

ii. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

This standard is a preceded by which of the following standards? *(Select one.)*

- ❍ a) Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
  - i. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
  - ii. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
- ❍ b) Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- ❍ c) Solve quadratic equations in one variable.
  - i. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  - ii. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- ❍ d) Solve quadratic equations with real coefficients that have complex solutions.
- ❍ e) I don't know

Correct Answers - a

PURPOSE

The Standards are designed around coherent progressions that develop across grades. Understanding how content builds and connects to grades before and after is critical for teachers to properly support all students. This question asks teachers to choose the closest related standard that would precede the standard in the box; the standard comes from a prior grade in a related domain.

RATIONALE

a) Using variables to represent quantities and constructing simple equations and inequalities to solve problems by reasoning about the quantities is conceptually a precursor to the standard in the box, in which students use this knowledge to begin solving linear equations in one variable.

b) Although very closely connected, solving linear equations in which coefficients are represented by letters

follows the standard in the box conceptually.
c) Solving quadratic equations in one variable would be presented after students learn to solve linear equations in one variable.
d) Solving quadratic equations with real coefficients that have complex solutions is even more advanced than solving quadratic equations in one variable and would also be presented after students learn to solve linear equations in one variable.

Q. 9

Shift of Coherence

Question
Consider this standard.

Solve quadratic equations in one variable.

i. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

ii. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

This standard prepares students for which of the following standards? *(Select one.)*

- ❍ a) Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
  - i. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
  - ii. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
- ❍ b) Solve linear equations in one variable.
  - i. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
  - ii. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- ❍ c) Solve quadratic equations with real coefficients that have complex solutions.
- ❍ d) Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- ❍ e) I don't know

Correct Answers - c

PURPOSE

This question asks teachers to choose the closest related standard that would follow the standard in the box. The correct answer comes from the next grade in a related domain.

RATIONALE

a) Using variables to represent quantities and constructing simple equations and inequalities to solve problems by reasoning about the quantities is conceptually a precursor to solving quadratic equations and therefore would not follow the standard in the box.
b) Solving linear equations in one variable would be presented before students learn to solve quadratic equations in one variable.
c) Solving quadratic equations with real coefficients that have complex solutions builds upon students' ability to solve quadratic equations in one variable by increasing the cognitive demand of the numbers involved.
d) Solving linear equations and inequalities in one variable, including equations by coefficients, would be presented before students learn to solve quadratic equations in one variable.

Q. 10

Shift of Coherence

Question

From the list of five standards below, choose three (in any order) that could be taught together in a coherent unit.

- ❒ a) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- ❒ b) Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- ❒ c) Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
- ❒ d) Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- ❒ e) Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- ❒ f) Derive the equation of a parabola given a focus and directrix.

Correct Answers - a, b, e

PURPOSE

Understanding how standards relate to one another and support each other is a powerful tool in the classroom to ensure each lesson is not treated as a discrete topic and students are given the opportunity to build upon their understanding and prior knowledge. This question asks teachers to identify coherence within a grade as opposed to across grades as seen in questions 8 and 9.

RATIONALE

The standards listed in a, b, and e all relate to understanding and solving equations and could reasonably be taught together in a coherent unit. The other standards listed in c, d, and f address polynomial identities, understanding that polynomials form a system, and the equation for a parabola which do not directly relate to understanding and solving equations.

a) Creating equations in two or more variables relates to both understanding what a graphed equation represents as well as solving systems of linear equations.
b) Understanding that graphing an equation is the set of all its solutions plotted relates to both creating and graphing equations and solving systems of linear equations.
c) Proving polynomial identities does not conceptually relate to the other standards listed.
d) Understanding that polynomials form a system does not relate to understanding and solving pairs of linear equations
e) Solving systems of linear equations directly relates to creating and graphing equations and understanding what a graphed equation represents.
f) The equation of a parabola does not relate to the other standards listed.

Q. 11
Shift of Coherence

Question
Please briefly explain your reasoning for your selections in Q. 10

PURPOSE
This question is not scored. It allows teachers to attach a rationale to their responses. We recommend teachers share their ideas and reflections on developing a coherent unit with their PLCs as a collaborative learning opportunity.

Q. 12
Shift of Focus

Question
Yesterday, Mr. Jones taught a math lesson to his high school class and he would like comments on a few elements of his lesson.

Mr. Jones wants to be sure his lesson plan was Common Core-aligned. This was the objective for his class: *"Students will sort and classify equations by carrying out algebraic manipulations and by substituting numbers into algebraic statements in order to test their validity in special cases."*

This objective is:

- ○ a) Supporting work of high school
- ○ b) Major work of high school
- ○ c) Not a high school objective
- ○ d) I don't know.

Correct Answers - b

PURPOSE

A large majority of class time should be devoted to the Major Work of the grade and Widely Applicable Prerequisites, as it is the content most central to grade-level understanding. The lesson objective is often used to convey the intent of the lesson, and it is frequently where information about the standard(s) in the lesson can be found. This question asks teachers to use the lesson objective to identify the standard in the lesson and identify how that standard relates to the focus of the grade (specifically, whether it is appropriate for the grade and whether it is Major Work). For more information, see Criterion #1 of the HS Publishers' Criteria for the Common Core at www.achievethecore.org/publisherscriteria-math-hs.

RATIONALE

The lesson objective refers to students carrying out algebraic manipulations and testing the validity of equations in special cases which is a Widely Applicable Prerequisite for college and career.

PC

Q. 13

Practice-Content Connections

Question

Which activity would be appropriate for this lesson objective? Select one.

- a) Students graph a collection of quadratic equations in vertex form.
- b) Students sort and classify equations as linear, exponential, and quadratic, and justify their results based on the graphs.
- c) Students algebraically manipulate linear equations from point-slope to slope-intercept form.
- d) Students create transformations of parabolas by changing the algebraic representation of the graph.
- e) I don't know.

Correct Answers - b

PURPOSE

Different standards require different types of activities or tasks depending upon what understanding or application a student needs to demonstate in order to exhibit mastery. It is important that any activity students are asked to do supports the learning and practice of the lesson's target. This question asks teachers to choose the appropriate lesson activities for a targeted standard.

RATIONALE

a) Graphing a collection of quadratic equations in vertex form would not meet the lesson objective of sorting and classifying equations to test their validity in special cases.
b) Sorting and clasifying equations as linear, exponential, and quadratic and justifying their results based on the

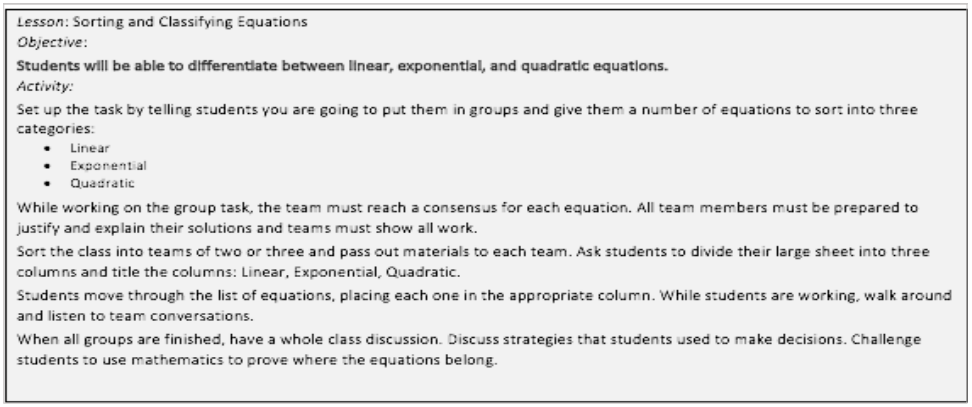
graphs would support the lesson objective.
c) Mainpulating linear equations to slope-intercept form does not relate to sorting and classifying equations to test their validity in special cases.
d) Creating transformations of parabolas does not relate to sorting and classifying equations to test their validity in special cases.

Q. 14

Shift of Rigor

Question

In the lesson plan below, which type(s) of student learning is/are addressed? Please select all that apply.



Lesson: Sorting and Classifying Equations
Objective:
Students will be able to differentiate between linear, exponential, and quadratic equations.
Activity:
Set up the task by telling students you are going to put them in groups and give them a number of equations to sort into three categories:

- Linear
- Exponential
- Quadratic

While working on the group task, the team must reach a consensus for each equation. All team members must be prepared to justify and explain their solutions and teams must show all work.
Sort the class into teams of two or three and pass out materials to each team. Ask students to divide their large sheet into three columns and title the columns: Linear, Exponential, Quadratic.
Students move through the list of equations, placing each one in the appropriate column. While students are working, walk around and listen to team conversations.
When all groups are finished, have a whole class discussion. Discuss strategies that students used to make decisions. Challenge students to use mathematics to prove where the equations belong.

Correct Answers - a

PURPOSE

It is important to identify the aspect(s) of rigor targeted in a standard in order to know what type of activities are most appropriate for the instructional materials. This question asks teachers to analyze the lesson plan and identify which aspect of rigor is addressed in the objective and activities.

RATIONALE

a) In the above lesson plan, the objective is for students to "be able to differentiate between linear, exponential, and quadratic equations" which requires students to understand the distinctions between these three types of equations and use this knowledge in an activity sorting equations.
b) Students may graph equations to determine how to categorize them which is somewhat procedural, they are doing so to sort and develop a stronger understanding of equations and their solutions.
c) Students are working strictly with given mathematical equations. They are not being asked to represent real world situations with mathematics and are not applying their mathematical knowledge in a real-world problem-solving situation.

Q. 15

Shift of Focus

Question

Which standard is addressed in the lesson plan?

- ❒ a) Know and apply the Binomial Theorem for the expansion of $(x + y)n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
- ❒ b) Identify zeroes of polynomials when suitable factorizations are available, and use the zeroes to construct a rough graph of the function defined by the polynomial.
- ❒ c) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- ❒ d) Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
- ❒ e) None of the above.
- ❒ f) I don't know.

Correct Answers - c

PURPOSE

Building lesson objectives from the language of the targeted standard(s) is critical to ensuring the lesson will meet the expectations of that standard. This question asks teachers to use the lesson objective to identify the standard addressed.

RATIONALE

a) Knowing and applying the Binomial Theorem does not directly relate to the lesson objective about classifying and sorting equations.
b) Identifying zeroes of polynomials does not relate to the lesson objective about classifying and sorting equations.
c) Graphing functions and showing key features of the graph aligns to the lesson objective because students understand and apply their knowledge of sorting and classifying equations as linear, quadratic, or exponential.
d) Solving application problems involving two linear equations in two variables does not directly relate to the lesson objective about classifying equations.

Q. 16
Rigor

Question
Briefly, how would you improve upon the lesson plan to more thoroughly address this/these standard(s)?

PURPOSE
This lesson was chosen because it offered specific sample material for the questions being asked, not because it is an exemplary lesson and should be used as a model. This question is not scored; it offers teachers an opportunity to submit revisions and reflect on improvements for the lesson. We recommend teachers share the ideas and reflections on the lesson with their PLCs as a collaborative learning opportunity.

Q. 17
Practice-Content Connections

Question
Given the sample lesson plan, which statement(s) below reflect the lesson? Please select all that apply.

- ❒ a) The lesson asks students to attend to precision.
- ❒ b) The lesson asks students to reason abstractly and quantitatively.
- ❒ c) The lesson asks students to construct arguments and critique reasoning in others.
- ❒ d) The lesson requires students to model with mathematics.
- ❒ e) The lesson has students look for and express repeated reasoning.
- ❒ f) The lesson builds on previous knowledge.
- ❒ g) The lesson encourages students to use appropriate tools strategically.
- ❒ h) I don't know.

Correct Answers - a, e, f

PURPOSE
The mathematical practices describe the varieties of expertise math teachers should seek to develop in their students. These student behaviors and actions are elicited by a teacher in the classroom within a lesson and in connection to the classroom norms. The content and/or approach of a particular lesson should support the development of particular practices. This question asks teachers for the practices required based on the description of the activities in the lesson plan.

RATIONALE
a) The lesson activities require students to attend to precision as they manipulate and classify equations, attending to the nuances of the representations.
b) The lesson activities do not focus on students reasoning abstractly or quantitatively.
c) Students are using evidence to explain their answers in the latter part of the lesson, but they are not

constructing arguments or critiquing reasoning,
d) The lesson activities do not require students to solve real-world problems.
e) Classifying the equations requires students to look for and express repeated reasoning.
f) The lesson activities build on previous student knowledge because they are using their understanding of different types of equations and their graphs to classify them.
g) The lesson activities do not require students to use appropriate tools strategically.

Q. 18

Shift of Coherence

Question
Which prerequisite content will prepare students for this lesson? Select one.

- a) Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
- b) Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- c) Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- d) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in zeros or eventually repeats.
- e) None of the above.
- f) I don't know.

Correct Answers - C

PURPOSE

Considering how the content of the standards connects to one another and builds across grades is important for understanding the coherence of the Standards and for supporting differentiation within the classroom. This question asks teachers to reflect on the content addressed in the lesson in order to identify which mathematical concepts students would need to understand prior to the lesson.

RATIONALE

a) This standard addresses unit rates and ratios of fractions and does not directly build towards manipulating and classifying equations.
b) This standard about adding and subtracting rational numbers does not directly build towards manipulating and classifying equations.
c) Understanding the definition of a linear function and its graph is a direct pre-cursor to classifying non-linear equations.
d) This standard about converting a rational number to decimal is not a direct pre-cursor to manipulating and classifying equations.

Q. 19

Shift of Coherence

Question

This lesson most directly prepares students to learn which of the following standards? (Select one.)

- a) Know and apply the Binomial Theorem for the expansion of $(x + y)n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
- b) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- c) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, hand h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
- d) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- e) None of the above.
- f) I don't know.

Correct Answers - b

PURPOSE

This question asks teachers to reflect on the content addressed in the lesson in order to identify which mathematical concepts students would be prepared to learn next.

RATIONALE

a) This standard addresses knowing and applying the Binomial Theorem and does not relate to classifying equations.
b) Classifying equations is a direct pre-cursor to graphing rational functions as it allows students to analyze the features of those functions.
c) This standard about composing functions is not as directly related as classing equations as is graphing rational functions.
d) This standard about understanding rational expressions form a system analogous to rational numbers does not directly relate to classifying equations.

Q. 20

Practice-Content Connections

Question

Thinking about the last complete unit that you taught, how often did you do the following?

	Never	Rarely	Sometimes	Often	In all or most lessons
a) Build on prior skills and knowledge when teaching new content					
b) Ground procedures and formulas in conceptual understanding					
c) Make the mathematics of the lesson explicit by using explanations, representations, and/or examples					
d) Use repeated practice to improve students' computational skills					
e) Have students do work with and practice grade-level problems and exercises. f) Emphasize one solution method to strengthen all students' understanding of the content					
g) Have students choose and use appropriate tools when solving a problem					
h) Check for understanding throughout the lesson using informal, but deliberate methods (such as questioning or assigning short problems)					
i) Summarize the mathematics with references to student work to reinforce the focus of the lesson					
j) Predominantly use questions and problems that are from the textbook					
k) Review standards from previous grades					
l) Ask students to explain and justify their work					
m) Provide feedback to help students revise initial work					

Correct Answers - N/A

PURPOSE

As with question 7, this question details an array of choices a teacher can make when supporting students with mathematics and asks teachers to identify which behaviors they exhibit in the classroom. Given the variety of decisions that can be made in different educational contexts, many of the statements above could be valid. However, some instructional practices better support the content and structure of the Standards, and a teacher fully implementing the Standards would be more likely to emphasize those practices over others. To learn more about Shifts-aligned instructional practice, go to www.achievethecore.org/instructional-practice.

RATIONALE

The practices associated with the italicized sections below are those that a teacher fully implementing the Standards is likely to emphasize.

a) *Building on previous work ensures that students see math as cohesive as opposed to a set of discrete skills.*
b) *Grounding procedures and formulas in conceptual understanding ensures that students understand why a procedure works and will be able to apply it to more complex mathematics.*

c) *Using a variety of explanations, representations, and examples to make the mathematics of the lesson clear helps to reinforce student understanding. Representations display concepts or problems by using drawings or models, whereas examples may show different strategies or methods to solve a particular problem.*
d) *Repeated practice is important for students to build towards the fluency and procedural skill expectations of their grade-level standards.*
e) *All students need consistent practice with grade-level problems and exercises in order to meet the expectations of the Shifts.*
f) A variety of student solution methods should be shared and examined together to support mathematical understanding for all students.
g) *MP.5 explicitly states that students should choose appropriate tools strategically. Students must have the opportunity to select tools, when appropriate.*
h) *Checks for understanding throughout the lesson allow a teacher to assess progress of all students and make adjustments to instruction, as needed.*
i) *A lesson that includes a summary with references to student work and discussion reinforces the mathematics of the lesson and supports student learning.*
j) All materials should be carefully reviewed for alignment to the Shifts. If the text is found to be aligned to the Shifts, using its tasks frequently is appropriate. However, if the text is not aligned to the Shifts, educators should explore supplementing with other resources for questions and problems.
k) While there may be appropriate times to briefly review the standards of a previous grade, the majority of instruction should be on grade-level standards.
l) *Asking students to regularly explain and justify work and providing feedback that helps students revise initial work supports student in developing mathematical proficiency.*
m) *Providing feedback on student work and allowing students to revise and correct/improve their answers supports students in perseverance and developing strong arguments and evidence in their mathematical reasoning.*

Q. 21

Practice-Content Connections

Question
A teacher walking around the classroom overhears the comments below during student group work. Which comment shows students demonstrating the practice standard "construct viable arguments and critique the reasoning of others"? Select one.

- a) "That could be the answer, or the answer could be $y = x^2 + 2$."
- b) "No, the answer can't be $y = -x^2 + 2$, because we know the parabola opens up so it must have a positive squared term."
- c) "Yep, I agree that the answer is $y = -x^2 - 2$."
- d) "I don't think the answer is $y = -x^2 - 2$. I'm going to ask the teacher."
- e) I don't know

Correct Answers - b

PURPOSE
Understanding how the mathematical practices are translated into the classroom and student behavior is important for ensuring CCSS-aligned instruction. This question asks teachers to select the activity that best embodies MP.3.
RATIONALE
a) The student's explanation does not include any reasoning.

b) The student clearly communicates reasoning in her response.
c) The student agrees with no justification or reasoning.
d) The student relies on the teacher for identifying and/or understanding the correct answer.

PC

Q. 22

Practice-Content Connections

Question

Which scenario shows students demonstrating the practice standard "**model with mathematics**"? Select one.

- ❍ a) Students completing a worksheet on factoring quadratics to identify the zeros.
- ❍ b) Students interpreting the graphs of polynomial expressions based on their coefficients.
- ❍ c) Students working together to create a function that represents the path of a kicked football.
- ❍ d) Students answering a set of routine word problems on quadratic relationships.
- ❍ e) I don't know

Correct Answers - c

PURPOSE

Understanding how the mathematical practices are translated into classroom activities and student behavior is important for ensuring aligned instruction. This question asks teachers to select the activity that best embodies MP.4.

PURPOSE

a) Students are performing procedural skill/fluency and are not applying mathematics in a real-world situation.
b) Students are expressing their understanding, but do not apply their knowledge of mathematics to a real-world situation
c) Students are using mathematics to solve a real-world problem.
d) Students are not solving unique problems that require the application of a variety of skills and knowledge.

Part 2 : Shifts Summary

In the following section, each category will be shown along with the questions from the Survey that comprise that data. These are followed by discussion questions and recommended resources.
The discussion questions are meant to provide guidance for coaches and teachers to collectively reflect on classroom practice in relation to the Shifts. We recommend that these be used in a professional learning community or professional development setting in which educators can have an open and honest dialogue about the current state of aligning instruction and practice to the Standards. These conversations will help educators continue to develop strategies and make progress towards effective implementation of the Shifts.

The recommended resources are meant to provide a starting point to support the discussion between instructional leaders/coaches and teachers, and to help educators continue to learn about the Shifts and how they can be translated into classroom practice. You can find these and many other professional development and classroom resources at www.achievethecore.org.

Category	Topic	Question Number
Focus	Identify Major Work of the Grade	4, 12, 15
	Agreement with the statements about dedicating time to the major work of the grade.	7a, 7j, 7g
Rigor	Identification of rigor targeted by a standard and how rigor manifests in a lesson plan.	6, 14
	Evaluate the quality of lessons based on aspects of rigor	16 (unscored)
	Agreement with the statements about balance teaching the three aspects of rigor throughout the year.	7b, 7c, 7e, 7i
Coherence	Understanding of the progression of standards within and across grades.	8, 9, 18, 19
	Recognize how standards in a grade form a coherent unit.	10, 11 (unscored)
	Frequency at which teachers meet with colleagues to discuss the Common Core State Standards for Mathematics.	5a, 5b
	Agreement with statements about using coherence to organize their curriculum.	7d, 7f, 7h, 7k
Practice-Content Connections	Frequency in which teachers connect the standards for mathematical practices and standards for mathematical content in the classroom.	20
	Identification of mathematical practices elicited in a lesson plan.	17, 13, 21, 22

Shift of Focus

DISCUSSION QUESTIONS

1. What does Major Work of the grade mean? Why is it so important?
2. What is the Major Work of our grade?
3. Approximately how much time are we teaching Major Work? What's our evidence?
4. How have we adapted our curricular materials to spend more time on the Major Work of the grade?
5. What have we stopped teaching since implementing the CCSS for Mathematics?
6. What have we taught less of since implementing the CCSS for Mathematics?

Resources

1. The Shifts (http://achievethecore.org/shifts-mathematics): Webpage containing information and resources of the three Shifts
2. Focus by Grade Level (http://achievethecore.org/focus): A collection of PDFs detailing the mathematical content emphasized in the Standards by grade level. Includes Widely Applicable Prerequisites for High School.

Shift of Coherence

DISCUSSION QUESTIONS

1. When do we work with teachers in surrounding grade levels to help students make connections across grades/courses?
2. What opportunities have we found to create coherence among the standards for our students, both within and across grades/courses?
3. Have we found ways to connect Major/Supporting Work in our grade/course? When? What are examples?

Resources

1. The Shifts (http://achievethecore.org/shifts-mathematics): Webpage containing information and resources of the three Shifts
2. Progressions Documents (http://achievethecore.org/progressions): A collection of narratives that explain how mathematical content develops coherently across grades

Shift of Rigor

DISCUSSION QUESTIONS

1. What are the fluency or procedural skill standards for our grade?
2. What standards in our grade focus on conceptual understanding (in which students make meaning of the math)?
3. Which standards in our grade ask students to apply their knowledge in real-world settings?
4. Are all students given time for regular practice with the fluency standards?
5. Does our teaching reflect a balance of conceptual understanding, procedural skill/fluency, and application? What is our evidence?

Resources

1. The Shifts (http://achievethecore.org/shifts-mathematics): Webpage containing information and resources of the three Shifts
2. Annotated Tasks (http://achievethecore.org/math-tasks): Math tasks that illustrate the K-12 standards
3. Annotated Mini-Assessments (http://achievethecore.org/math-mini-assessments): A collection of mini-assessments designed for teachers to use

Practice-Content Connections

DISCUSSION QUESTIONS

1. Are we incorporating the mathematical practices in a way that ensures students learn grade-level content?
2. How do we make decisions about which mathematical practices to target within a specific lesson?
3. What evidence do we look for from students to know that they are demonstrating the mathematical practices?

Resources

1. The Standards for Mathematics Practice (http://achievethecore.org/math-practices): excerpted from the Standards, describe the behaviors and skills meant to be elicited by teachers in the math classroom.
2. Planning Tool (http://achievethecore.org/planning-tool): Lesson Planning Tool
3. Coaching Tool (http://achievethecore.org/coaching-tool): Tool to assist teachers, and those who support them, build understanding about CCSS-aligned instruction